Homework 2 Solutions

Problem 1

S_y = 350 MPa.

MSS: $\sigma_1 - \sigma_3 = S_y / n \quad \Rightarrow \quad n = \dfrac{S_y}{(\sigma_1 - \sigma_3)}$

DE: $\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = \dfrac{S_y}{n} \quad \Rightarrow \quad n = \dfrac{S_y}{\sigma'}$

(a) MSS: $\sigma_1 = 100$ MPa, $\sigma_3 = 0 \;\Rightarrow\; n = \dfrac{350}{100-0} = 3.5$ *Ans.*

DE: $n = \dfrac{350}{[100^2 - (100)(100) + 100^2]^{1/2}} = 3.5$ *Ans.*

(b) MSS: $\sigma_1 = 100$, $\sigma_3 = -100$ MPa $\;\Rightarrow\; n = \dfrac{350}{100-(-100)} = 1.75$ *Ans.*

DE: $n = \dfrac{350}{\left[100^2 - (100)(-100) + (-100)^2\right]^{1/2}} = 2.02$ *Ans.*

(**c**) MSS: $\sigma_1 = 0$, $\sigma_3 = -100$ MPa $\;\Rightarrow\; n = \dfrac{350}{0-(-100)} = 3.5$ *Ans.*

DE: $n = \dfrac{350}{\left[(-50)^2 - (-50)(-100) + (-100)^2\right]^{1/2}} = 4.04$ *Ans.*

Problem 2

S_{yt} = 60 kpsi, S_{yc} = 75 kpsi. Eq. (5-26) for yield is

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}}\right)^{-1}$$

(**a**) $\sigma_1 = 25$ kpsi, $\sigma_3 = 0 \quad \Rightarrow \quad n = \left(\dfrac{25}{60} - \dfrac{0}{75}\right)^{-1} = 2.40$ *Ans.*

(**b**) $\sigma_A, \sigma_B = \dfrac{-12+15}{2} \pm \sqrt{\left(\dfrac{-12-15}{2}\right)^2 + (-9)^2} = 17.7, -14.7$ kpsi

$\sigma_1 = 17.7$, $\sigma_2 = 0$, $\sigma_3 = -14.7$ kpsi $\quad \Rightarrow \quad n = \left(\dfrac{17.7}{60} - \dfrac{-14.7}{75}\right)^{-1} = 2.04$ *Ans.*

Problem 3

S_{ut} = 30 kpsi, S_{uc} = 90 kpsi

BCM: Eqs. (5-31), p. 250
MM: see Eqs. (5-32), p. 250

(**a**) $$\sigma_A,\ \sigma_B = \frac{-15+10}{2} \pm \sqrt{\left(\frac{-15-10}{2}\right)^2 + (-15)^2} = 17.03,\ -22.03 \text{ kpsi}$$

BCM: Eq. (5-31*b*), $$n = \left(\frac{17.03}{30} - \frac{-22.03}{90}\right)^{-1} = 1.23 \quad \textit{Ans.}$$

MM: $\sigma_A \geq 0 \geq \sigma_B$, and $|\sigma_B/\sigma_A| \geq 1$, Eq. (5-32*b*),

$$n = \left[\frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc}S_{ut}} - \frac{\sigma_B}{S_{uc}}\right]^{-1} = \left[\frac{(90-30)17.03}{90(30)} - \frac{-22.03}{90}\right]^{-1} = 1.60 \quad \textit{Ans.}$$

(**b**) σ_A = 15 kpsi, σ_B = –15 kpsi,

BCM: Eq. (5-31*a*), $$n = \left(\frac{15}{30} - \frac{-15}{90}\right)^{-1} = 1.5 \quad \textit{Ans.}$$

MM: $\sigma_A \geq 0 \geq \sigma_B$, and $|\sigma_B/\sigma_A| \leq 1$, Eq. (5-32a), $$n = \frac{S_{ut}}{\sigma_A} = \frac{30}{15} = 2.0 \quad \textit{Ans.}$$

Problem 4

From Table A-20, S_y = 370 MPa. From the solution of Prob. 3-69, in the plane of analysis

σ_1 = 275 MPa, σ_2 = – 12.1 MPa, and τ_{max} = 144 MPa

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 275 MPa, σ_2 = 0, and σ_3 = – 12.1 MPa

MSS: From Eq. (5-3), $$n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{370}{275-(-12.1)} = 1.29 \qquad \textit{Ans.}$$

DE: From Eqs. (5-13) and (5-19)

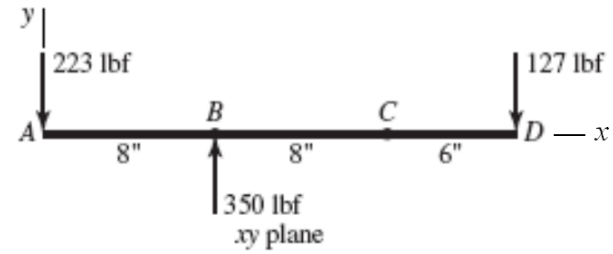
$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{370}{\left[275^2 - 275(-12.1) + (-12.1)^2\right]^{1/2}}$$

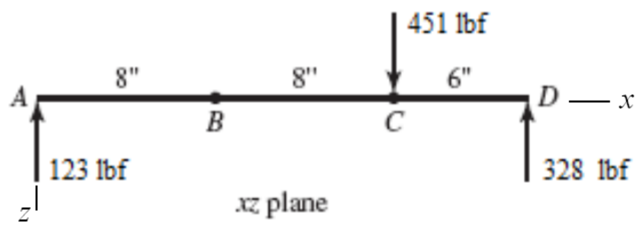
$$= 1.32 \qquad \textit{Ans.}$$

Problem 5

From Table A-20, for AISI 1035 CD, S_y = 67 kpsi.

From force and bending-moment equations, the ground reaction forces are found in two planes as shown.





The maximum bending moment will be at *B* or *C*. Check which is larger. In the *xy* plane,

$M_B = 223(8) = 1784 \text{ lbf} \cdot \text{in}$ and $M_C = 127(6) = 762 \text{ lbf} \cdot \text{in}$.

In the *xz* plane, $M_B = 123(8) = 984 \text{ lbf} \cdot \text{in}$ and $M_C = 328(6) = 1968 \text{ lbf} \cdot \text{in}$.

$$M_B = [(1784)^2 + (984)^2]^{\frac{1}{2}} = 2037 \text{ lbf} \cdot \text{in}$$

$$M_C = [(762)^2 + (1968)^2]^{\frac{1}{2}} = 2110 \text{ lbf} \cdot \text{in}$$

So point *C* governs. The torque transmitted between *B* and *C* is *T* = (300 – 50)(4) = 1000 lbf·in. The stresses are

$$\tau_{xz} = \frac{16T}{\pi d^3} = \frac{16(1000)}{\pi d^3} = \frac{5093}{d^3} \text{ psi}$$

$$\sigma_x = \frac{32M_C}{\pi d^3} = \frac{32(2110)}{\pi d^3} = \frac{21\,492}{d^3} \text{ psi}$$

For combined bending and torsion, the maximum shear stress is found from

$$\tau_{\max} = \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xz}^2\right]^{1/2} = \left[\left(\frac{21.49}{2d^3}\right)^2 + \left(\frac{5.09}{d^3}\right)^2\right]^{1/2} = \frac{11.89}{d^3} \text{ kpsi}$$

Max Shear Stress theory is chosen as a conservative failure theory. From Eq. (5-3)

$$\tau_{\max} = \frac{S_y}{2n} = \frac{11.89}{d^3} = \frac{67}{2(2)} \qquad \Rightarrow \qquad d = 0.892 \text{ in} \qquad \textit{Ans.}$$